# RSHS SCHOOL

Yr 11/12

**Sems 2 Examination, 2012**

**Question/Answer Booklet**

# MATHEMATICS 2C/2D
**Section Two:**
**Calculator-assumed**

# SOLUTIONS

Student Number: In figures | | | | | | | | |

In words

Your name MARKING KEY

## Time allowed for this section
Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

## Materials required/recommended for this section
***To be provided by the supervisor***
This Question/Answer Booklet
Formula Sheet (retained from Section One)

***To be provided by the candidate***
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination.

## Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 50 | 33 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 100 | 67 |
| | | | **Total** | 150 | 100 |

## Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2012*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that you **do not use pencil**, except in diagrams.

See next page

**Section Two: Calculator-assumed** **(100 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

---

**Question 8** **(7 marks)**

(a) Write down a recursive rule for the arithmetic sequence 35, 29, 23, 17, ... (2 marks)

$T_{n+1} = T_n - 6 \ , \ T_1 = 35$ ✓ ✓

(b) A different sequence is defined by $T_{n+1} = 0.25 \times T_n$, $T_1 = 768$.

(i) Write down the first three terms of this sequence. (2 marks)

768, 192, 48 ✓ ✓

(ii) Write down $T_7$. (1 mark)

0.1875 ✓

(iii) Calculate $T_9$, rounded to 3 significant figures. (2 marks)

$T_9 = 0.01171875$ ✓
$\approx 0.0117$ to 3 sf ✓

**Question 9** **(7 marks)**

The cost per unit of gas used by residential customers in a city depends upon their average daily consumption, as shown in the table below.

| Residential Customers | Prices including GST |
|---|---|
| Supply charge | 18.54 cents per day |
| The first 12 units used on average per day | 12.79 cents per unit |
| Over 12 units used on average per day | 11.54 cents per unit |

(a) A consumer used an average of 9 units of gas per day over a period of 91 days.

(i) Calculate the supply charge for a period of 91 days. (1 mark)

$91 \times 0.1854 = \$16.87$ ✓

(ii) Calculate the cost of using 9 units per day for 91 days. (1 mark)

$91 \times 9 \times 0.1279 = \$104.75$ ✓

(iii) Calculate the total of the supply charge and gas cost for this consumer. (1 mark)

$16.87 + 104.75 = \$121.62$ ✓

(b) Another consumer used a total of 900 units of gas over a period of 60 days.

(i) How many units did they use on average per day? (1 mark)

$900 \div 60 = 15$ ✓

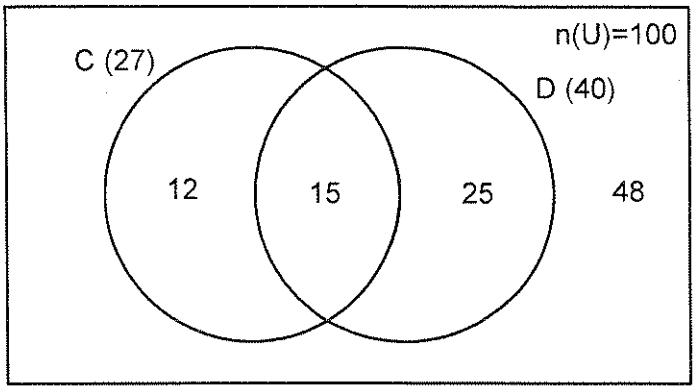
(ii) Calculate the total of the supply charge and gas cost for this consumer. (3 marks)

$60 \times 0.1854 = \$11.12$ ✓
$60 \times 12 \times .1279 = \$92.09$
$60 \times 3 \times .1154 = \$20.77$ ✓
$\text{Total} = \$123.98$ ✓

**Question 10** **(8 marks)**

In a recent survey of Australian households (the universal set U), it was noted that 40% own dogs (set D), 27% own cats (set C) and 48% own neither.

(a) Express this information using a Venn diagram, completing any missing entries. (3 marks)




✓ for number 15
✓ for 48
✓ for 25

(b) What percentage of households owned both dogs and cats? (1 mark)

15% ✓ follow through

(c) Describe the pet ownership of members of the set $\overline{C} \cap D$. (1 mark)

Households owning dogs but not cats. ✓

(d) If a household is chosen at random, determine

(i) $P(D \cup C)$ (1 mark)

52% ✓

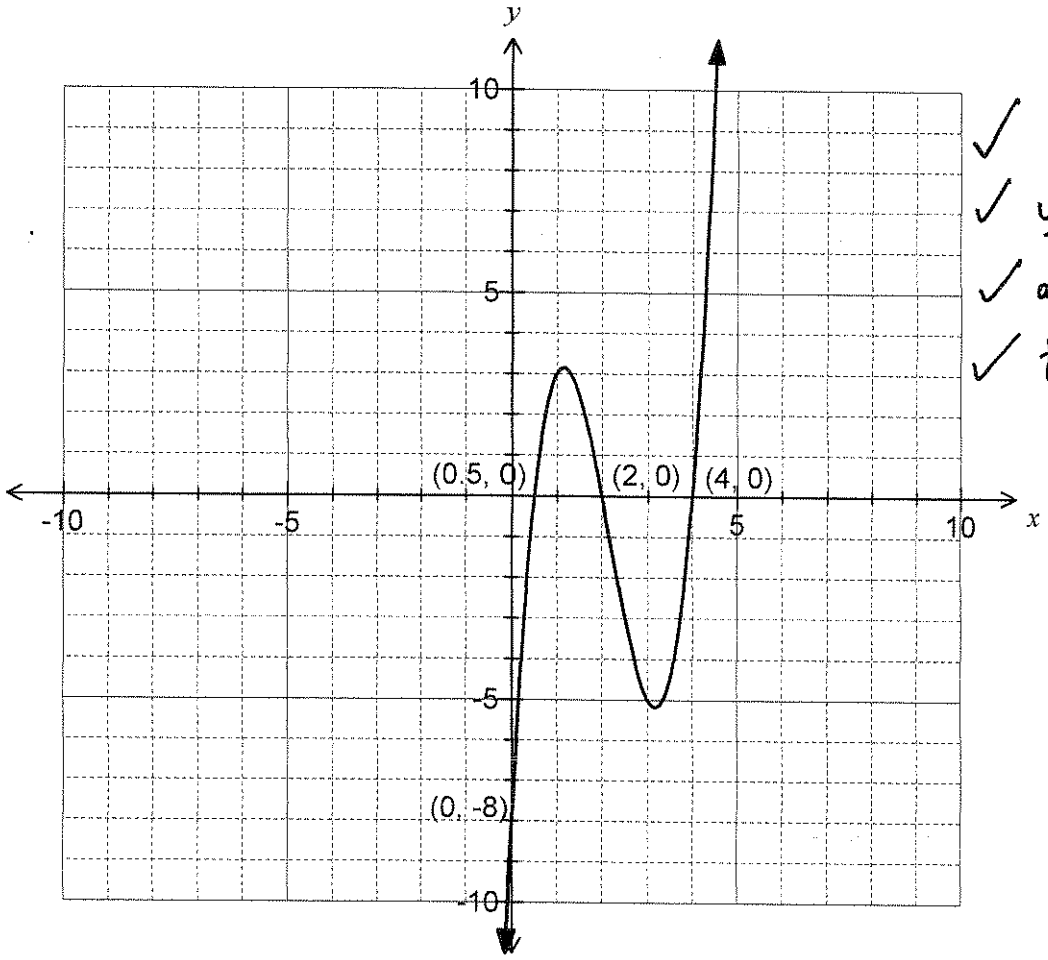
(ii) the probability that they own a dog given that they own a cat. (2 marks)

$\frac{15}{27}$ ✓ ✓

**Question 11** **(4 marks)**

On the axes below, draw the graph of $y = 2x^3 - 13x^2 + 22x - 8$, labelling all axes intercepts.

(4 marks)

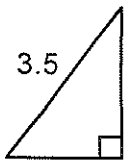



✓ shape
✓ y intercept
✓ all 3 x intercepts
✓ turning pts drawn clearly but not labelled

**Question 12** **(9 marks)**

(a) A straight ladder, 3.5 m long, stands on level ground and leans against a vertical wall so that it reaches 2.8 m up the wall.

(i) Sketch a diagram of the ground, wall and ladder, showing all given measurements. (1 mark)



✓

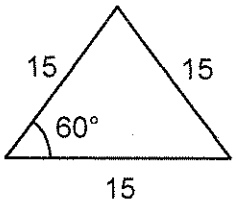
(ii) Calculate the angle that the ladder makes with the ground. (2 marks)

$$\sin\theta = \frac{2.8}{3.5}$$
$$\theta = 53°$$

✓ ✓ full marks for answer only

(b) The two ends of a length of rope are tied together to make a loop of length 45 m. The loop is then stretched out tightly on level ground to form a triangle.

(i) Calculate the area of this triangle, if it has three equal sides. (3 marks)

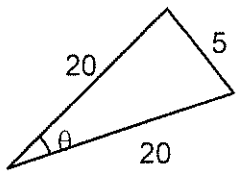



Angles must all be 60°

$A = 0.5(15)(15)\sin 60°$ ✓✓

$= 97.4\ m^2$ ✓

(ii) Calculate the size of the smallest angle in the triangle, if two of the sides are both 20 m long. (3 marks)



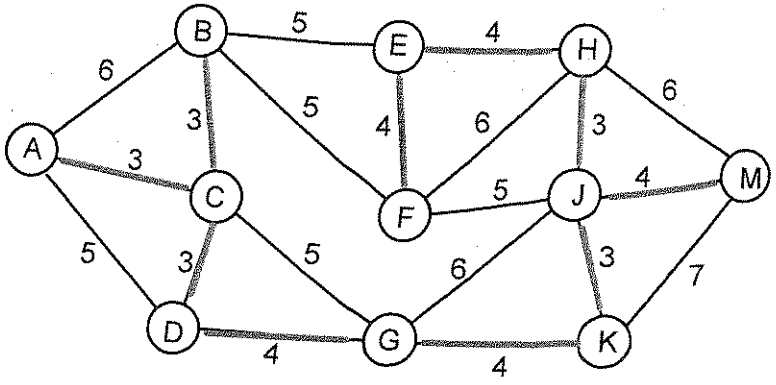

Smallest side opposite 5 m length ✓ idea (does not have to be stated)

$$\cos\theta = \frac{20^2 + 20^2 - 5^2}{2(20)(20)}$$ ✓

$\theta = 14°$ ✓

**Question 13** **(11 marks)**

Each number on the network below represents the distance, in kilometres, between adjacent vertices.




(a) Find the length of the minimum spanning tree of the network above, clearly indicating the tree on the diagram. (3 marks)

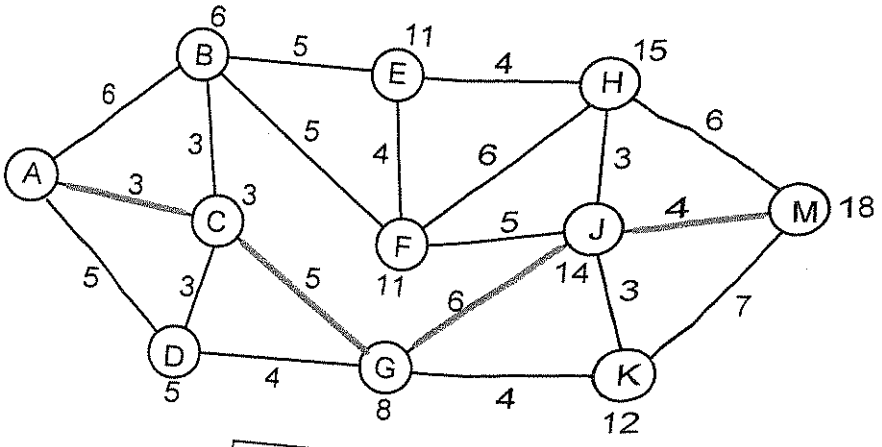
Length = 35 km

✓ at least 8 segments labelled correctly

✓ at least 9 segments correct

✓ all segments correct with 35 km stated.

(b) The network below is a copy of the one above. Determine the length of the shortest path from A to M and list the vertices passed through. To obtain full marks, numbers must be added to the diagram to show that a systematic method has been used. (4 marks)




Length = 18 km

✓ min totals (Any number stated for each node.

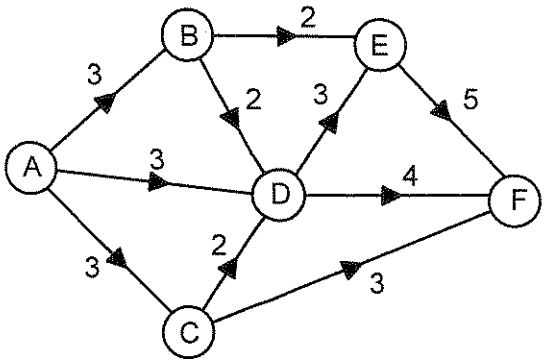
✓ at least 8 totals correct

✓ 18 stated near node M

✓ Length stated as 18 km.

See next page

(c) The following network shows the maximum number of daily flights between airports A to F in an easterly direction.




If each flight can carry 300 passengers, what is the maximum number of passengers that can be carried from A to F in a day? Systematic working must be shown to be awarded full marks. (4 marks)

ABEF = 2
ABDEF = 1
ADEF = 2 ✓✓
ADF = 1
ACF = 3

Total = 9 flights ✓

$9 \times 300 = 2700$ passengers per day ✓

follow through

**Question 14** **(11 marks)**

A newspaper contained an advertisement for a laptop computer with a cash price of $999.

(a) The computer could also be bought on credit over three years by weekly payments of $11.92, subject to approval.

(i) Assuming there are 52 weeks per year, how much would the computer cost if bought on credit? (1 mark)

$$11.92 \times 52 \times 3 = \$1859.52$$ ✓

(ii) How much interest is paid using the credit terms? (Assume that the interest is the extra amount paid when buying the computer on credit instead of cash). (1 mark)

$$1859.52 - 999 = \$860.52$$ ✓

(iii) Calculate the simple interest rate that would generate the interest calculated in (ii) on the cash price over the credit period. (3 marks)

✓ ✓

$$860.52 = \frac{999 \times R \times 3}{100}$$

$$R = 28.7\%$$ ✓

The same newspaper carried another advertisement from a lender offering unsecured loans for amounts up to $3000 at a rate of 15.9%pa with interest compounded monthly.

(b) How much interest would accumulate on a loan of $999 over three years, if no repayments were made? (3 marks)

$$F = 999 \times \left(1 + \frac{15.9}{100 \times 12}\right)^{12 \times 3}$$ ✓

$$= 1604.59$$ ✓

$$I = 1604.59 - 999$$

$$= \$605.59$$ ✓

follow through

See next page

(c) The recursive formula $T_{n+1} = 0.75T_n$ , $T_0 = 999$ can be used to calculate the value of the laptop computer after $n$ years.

(i) What annual rate of depreciation does this formula use? (1 mark)

$100 \times (1 - 0.75) = 25\%$ ✓

(ii) Calculate the value of the computer after five years, rounded to the nearest \$10. (2 marks)

$T_5 = 237.07$ ✓✓
$\approx \$240$ to nearest \$10

No need to round.
(Rounding has been assessed elsewhere)

**Question 15** **(12 marks)**

An electronics student built a circuit and display to generate random numbers from 0 to 99. The circuit designer claimed that all numbers had an equal chance of being chosen.

The first seven numbers generated by the circuit were 12, 5, 41, 64, 37, 2 and 23.

(a) For these seven numbers,

(i) write down the median (1 mark)

23 ✓

(ii) write down the range (1 mark)

$64 - 2 = 62$ ✓

(iii) explain why it does not make sense to calculate the mode. (1 mark)

None of the seven numbers occur more frequently than any of the others. ✓

To thoroughly test the circuit, 1000 numbers were generated and summarised in the table below.

| Number | 0-9 | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 | 80-89 | 90-99 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 167 | 172 | 151 | 121 | 97 | 74 | 85 | 51 | 47 | 35 |

(b) For these 1000 numbers,

(i) calculate the mean (2 marks)

$\bar{x} = 36.03$ ✓✓

(ii) calculate the standard deviation (1 mark)

Any number of decimal places

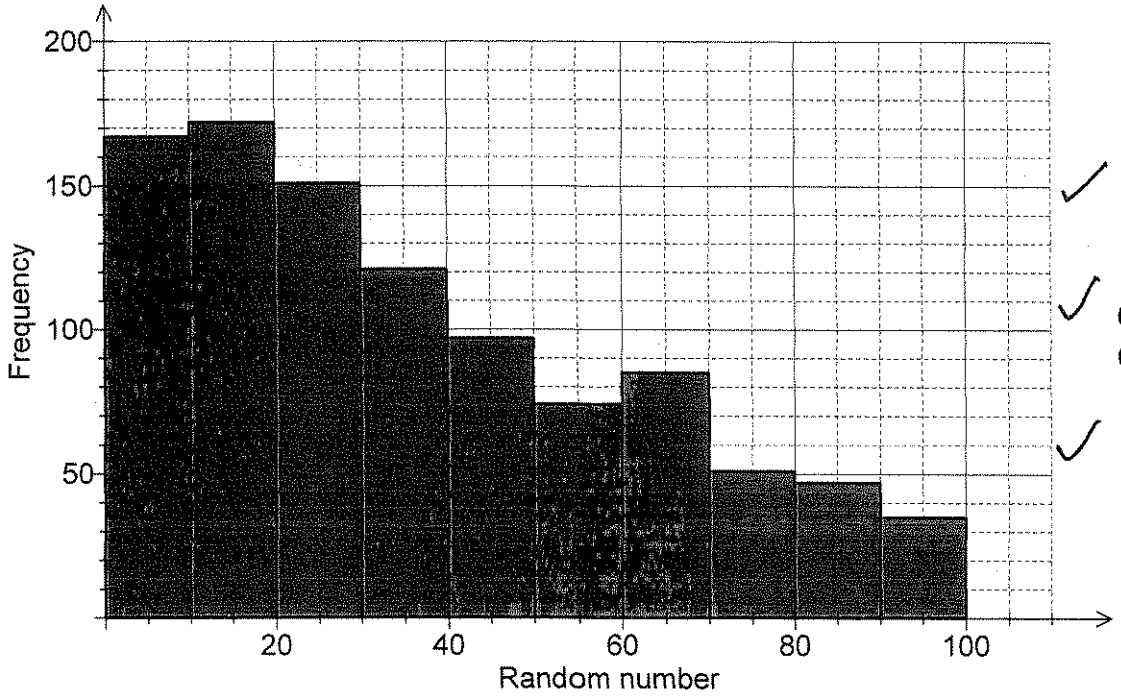
$sd = 25.94$ ✓

(iii) write down the modal group. (1 mark)

$10 - 19$ ✓

See next page

(c) Construct a frequency histogram for the 1000 numbers on the axes below. (3 marks)




✓ histogram form
✓ at least 6 freqs correct
✓ at least 8 freqs correct

(d) Use the above information to state whether the circuit designer's claim, that all numbers have an equal chance of being chosen, is justified. Explain your answer. (2 marks)

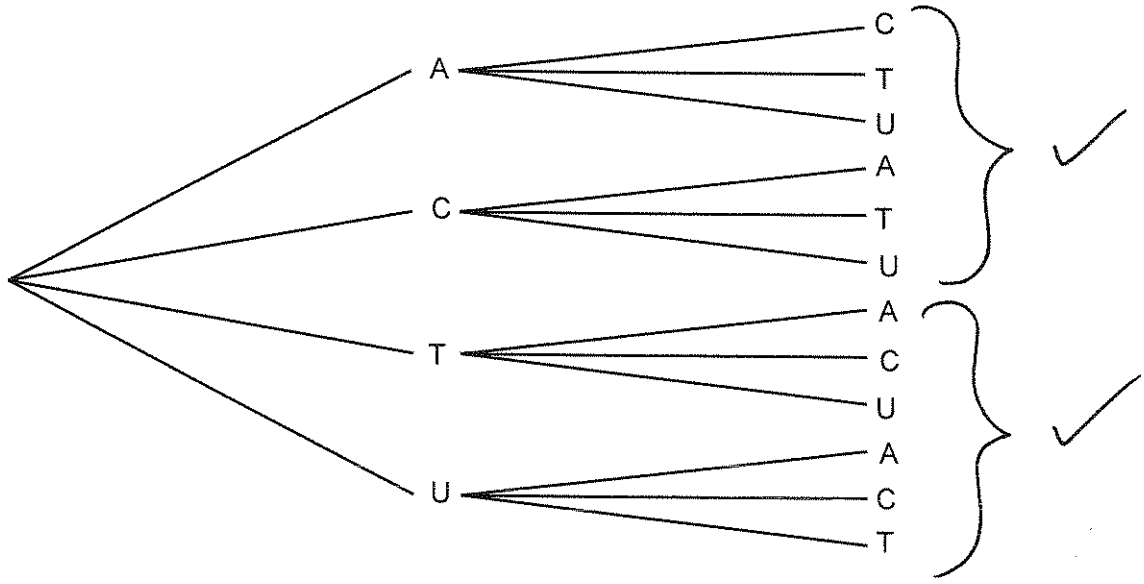
No. ✓
If all numbers had an equal chance of being chosen then one would expect roughly equal numbers in each group for such a large number of trials, but it can be seen that this is not the case. ✓

**Question 16** **(7 marks)**

A student wants to watch two different movies over the weekend, one on Saturday and one on Sunday. They have four movies to choose from, listed in the table below:

| Movie | Director | Year Released | Rating |
|---|---|---|---|
| Avatar | James Cameron | 2009 | PG |
| Chinatown | Roman Polanski | 1974 | M |
| Tess | Roman Polanski | 1979 | PG |
| Up | Pete Docter | 1979 | PG |

(a) Draw a tree diagram to show all the different orders they could watch two of these movies over the weekend. (2 marks)




(b) Given that all orders are equally likely, what is the probability that the student watches

(i) 'Avatar' on Saturday and 'Tess' on Sunday? (1 mark)

follow through

$\frac{1}{12}$

(ii) only movies directed by Roman Polanski? (1 mark)

CT or TC $= \frac{2}{12}$

(iii) at least one movie released in the 1970's? (1 mark)

1

(iv) the movie 'Up', given that both movies they watched were rated PG? (2 marks)

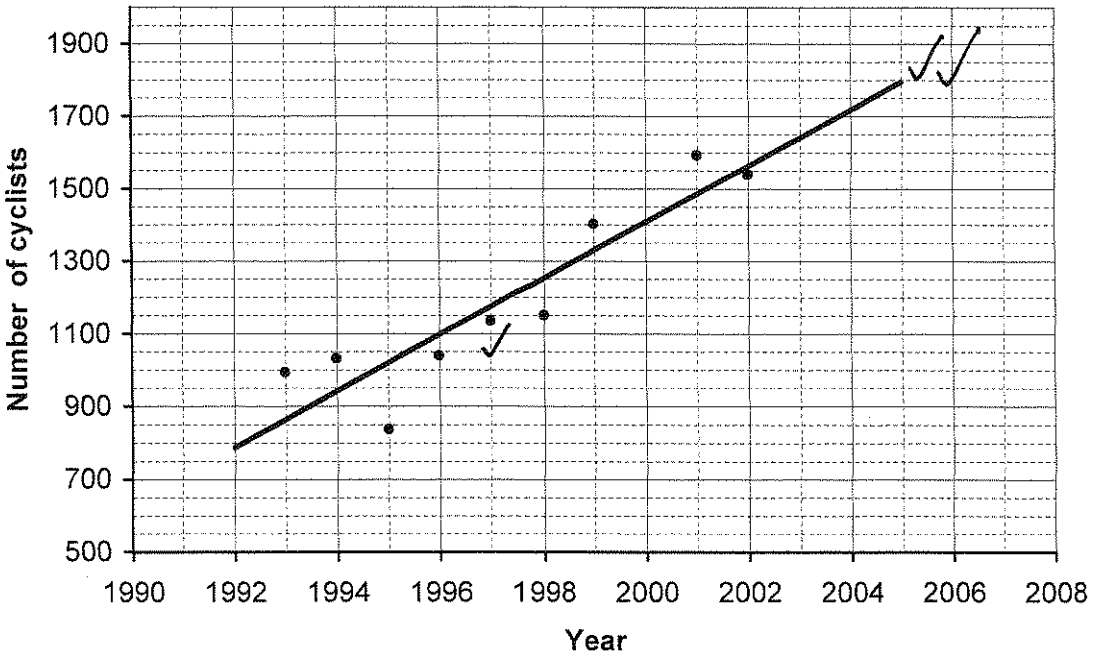
$\frac{4}{6}$

See next page

**Question 17** **(8 marks)**

The graph below shows the average weekday cyclist traffic flow on Bridge Road from 1993 to 2002. Data was not available for 1997 and 2000.

**Average weekday cyclist traffic flow on Bridge Road, 1993 - 2002**




(a) What was the average weekday cyclist traffic flow in 1994? (1 mark)

1030 ✓

(b) The average weekday cyclist traffic flow in 1997 was 1135. Add this point to the graph. (1 mark)

See graph above

(c) Draw a trend line on the graph. (2 marks)

(d) Use your trend line to estimate the average weekday cyclist traffic flow in:

(i) 2000 (1 mark)

1410 ✓ or near

(ii) 2004 (1 mark)

1720 ✓

(e) Comment, with reasons, on the reliability of each of your predictions in (d). (2 marks)

Prediction for 2000 is reliable as is interpolated and relationship is strong.
Prediction for 2004 is not reliable as is extrapolated. ✓

See next page

**Question 18** **(6 marks)**

A motoring club is planning an event and wants to know what its members would like. They plan to ask 80 members, chosen at random, from the total membership of 685 people.

(a) An extract from a table of three-digit random numbers is shown below:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 940 | 517 | 279 | 109 | 794 | 984 | 096 | 085 | 781 | 109 |

(i) Choose one of the numbers from the list and explain how the club could use it to select one of their members. (2 marks)

517 - Number the members from 1 to 685 and then pick member number 517. ✓ ✓

(ii) How many numbers in the above list could be used to select a sample? Explain your answer. (2 marks)

5 ✓

Six of the numbers are no more than 685, but one (109) is repeated, which leaves five to use. ✓

Note : Other explanations with correct reasoning may be accepted.

(b) One of the questions that the club plans to ask is the age of the member. They know that the youngest is 17 and that the oldest member is 63. Two possible data summary tables are shown below:

| Design A | |
|---|---|
| Age | Frequency |
| 15 - 25 | |
| 25 - 35 | |
| 35 - 45 | |
| 45 - 55 | |
| 55 - 65 | |

| Design B | |
|---|---|
| Age | Frequency |
| 10 - 19 | |
| 20 - 59 | |
| 60 - 69 | |

(i) State one disadvantage of using Design A. (1 mark)

Not clear into which age group a member aged 25, 35, etc would go. ✓

(ii) State one disadvantage of using Design B. (1 mark)

Too few groups - likely that most members would go into the middle group. ✓

See next page

**Question 19** **(10 marks)**

(a) $A$ and $t$ are related by the formula $A = 500(1.072)^t$.

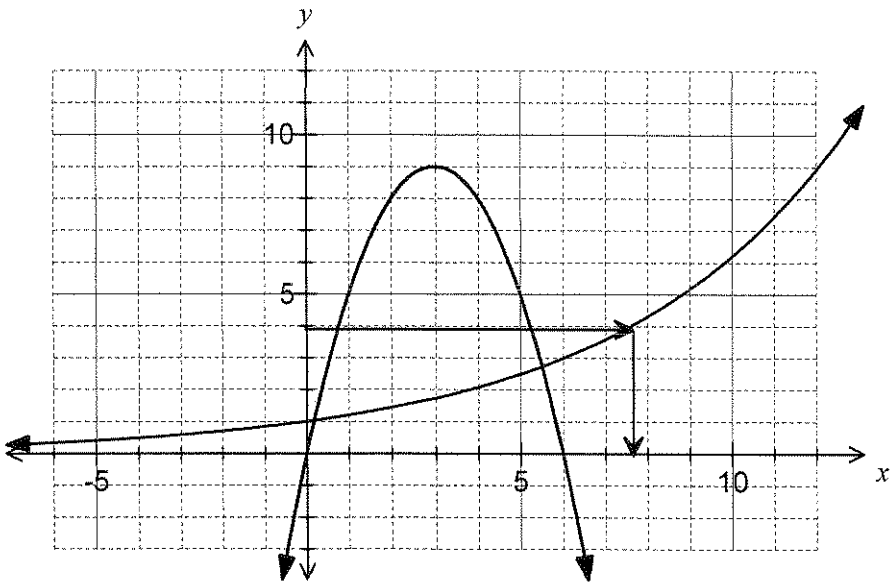
(i) Evaluate $A$ when $t = 5$, 6, and 7. (2 marks)

$500(1.072)^5 \approx 707.9$ ✓
$500(1.072)^6 \approx 758.8$
$500(1.072)^7 \approx 813.5$ ✓

(ii) Estimate the solution to $500(1.072)^t = 800$ if $t$ must be a whole number. (1 mark)

$t = 7$ ✓

(b) The graph of $y = 1.2^x$ is drawn below.




(i) Show use of the graph to solve the equation $1.2^x = 4$. (2 marks)

$x = 7.6$ ✓✓

One for answer
One for line drawn on graph above

(ii) Add the graph of $y = x(6 - x)$ to the axes above. (3 marks)

✓ shape ∩
✓ both $x$ intercepts
✓ Symmetry around $x = 3$

(iii) Explain how to use the two graphs above to solve the equation $x(6 - x) = 1.2^x$, and state how many solutions there are. There are no marks for solving the equation. (2 marks)

Read from the $x$-axis the $x$-coordinates of the points of intersection of the two curves. There are 2 solutions for $x$. ✓

**End of questions**

**Additional working space**

Question number: _____

**Additional working space**

Question number: _____